

## Volume Of Revolution - Washer Method

[www.mymathscloud.com](http://www.mymathscloud.com)

Questions in past papers often come up combined with other topics.  
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

Scan the QR code(s) or click the link for instant detailed model solutions!

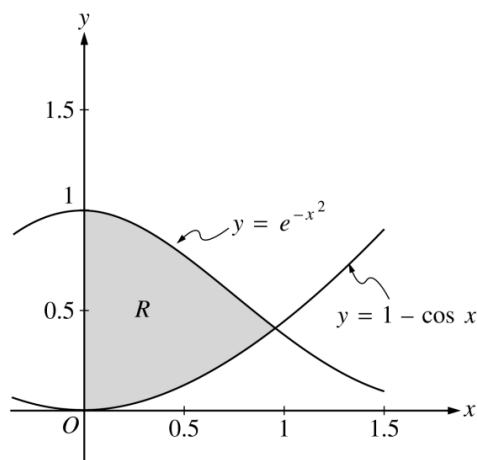
## Question 1

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Volume using Cross Sections, Integration - Area Between Curves, Volume of Revolution – Washer Method, Integration Technique – Trigonometry, Integration Technique – Exponentials

Paper: Part A-Calc / Series: 2000 / Difficulty: Easy / Question Number: 1



1. Let  $R$  be the shaded region in the first quadrant enclosed by the graphs of  $y = e^{-x^2}$ ,  $y = 1 - \cos x$ , and the  $y$ -axis, as shown in the figure above.
  - (a) Find the area of the region  $R$ .
  - (b) Find the volume of the solid generated when the region  $R$  is revolved about the  $x$ -axis.
  - (c) The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a square. Find the volume of this solid.

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

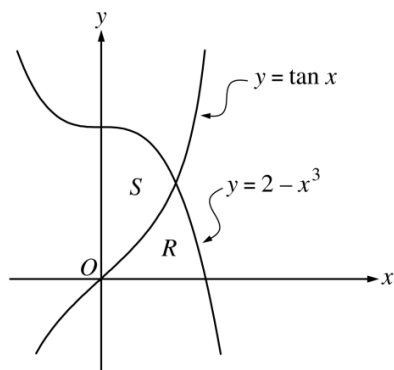
## Question 2

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method

Paper: Part A-Calc / Series: 2001 / Difficulty: Easy / Question Number: 1



1. Let  $R$  and  $S$  be the regions in the first quadrant shown in the figure above. The region  $R$  is bounded by the  $x$ -axis and the graphs of  $y = 2 - x^3$  and  $y = \tan x$ . The region  $S$  is bounded by the  $y$ -axis and the graphs of  $y = 2 - x^3$  and  $y = \tan x$ .
- Find the area of  $R$ .
  - Find the area of  $S$ .
  - Find the volume of the solid generated when  $S$  is revolved about the  $x$ -axis.

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

## Question 3

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Global or Absolute Minima and Maxima, Differentiation Technique – Exponentials

Paper: Part A-Calc / Series: 2002 / Difficulty: Somewhat Challenging / Question Number: 1

1. Let  $f$  and  $g$  be the functions given by  $f(x) = e^x$  and  $g(x) = \ln x$ .
- (a) Find the area of the region enclosed by the graphs of  $f$  and  $g$  between  $x = \frac{1}{2}$  and  $x = 1$ .
- (b) Find the volume of the solid generated when the region enclosed by the graphs of  $f$  and  $g$  between  $x = \frac{1}{2}$  and  $x = 1$  is revolved about the line  $y = 4$ .
- (c) Let  $h$  be the function given by  $h(x) = f(x) - g(x)$ . Find the absolute minimum value of  $h(x)$  on the closed interval  $\frac{1}{2} \leq x \leq 1$ , and find the absolute maximum value of  $h(x)$  on the closed interval  $\frac{1}{2} \leq x \leq 1$ . Show the analysis that leads to your answers.

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

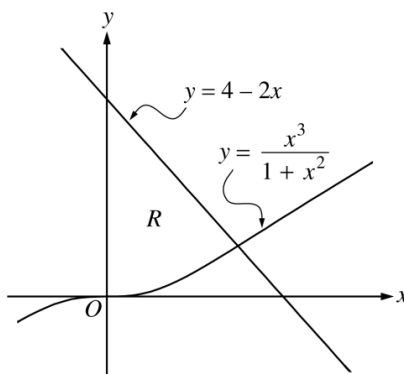
## Question 4

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2002-Form-B / Difficulty: Easy / Question Number: 1



1. Let  $R$  be the region bounded by the  $y$ -axis and the graphs of  $y = \frac{x^3}{1+x^2}$  and  $y = 4 - 2x$ , as shown in the figure above.
- Find the area of  $R$ .
  - Find the volume of the solid generated when  $R$  is revolved about the  $x$ -axis.
  - The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a square. Find the volume of this solid.

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

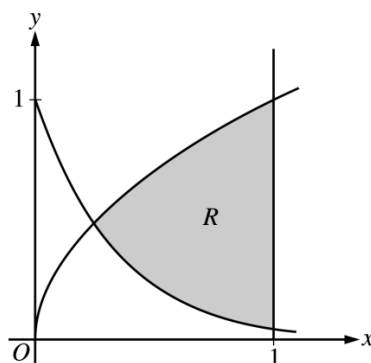
## Question 5

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2003 / Difficulty: Easy / Question Number: 1



1. Let  $R$  be the shaded region bounded by the graphs of  $y = \sqrt{x}$  and  $y = e^{-3x}$  and the vertical line  $x = 1$ , as shown in the figure above.
  - (a) Find the area of  $R$ .
  - (b) Find the volume of the solid generated when  $R$  is revolved about the horizontal line  $y = 1$ .
  - (c) The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a rectangle whose height is 5 times the length of its base in region  $R$ . Find the volume of this solid.

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

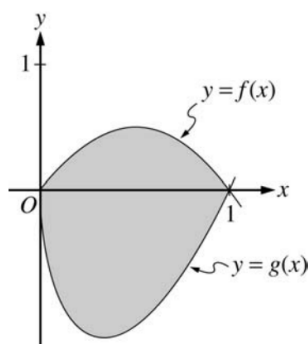
## Question 6

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2004 / Difficulty: Somewhat Challenging / Question Number: 2



2. Let  $f$  and  $g$  be the functions given by  $f(x) = 2x(1 - x)$  and  $g(x) = 3(x - 1)\sqrt{x}$  for  $0 \leq x \leq 1$ . The graphs of  $f$  and  $g$  are shown in the figure above.
- (a) Find the area of the shaded region enclosed by the graphs of  $f$  and  $g$ .
  - (b) Find the volume of the solid generated when the shaded region enclosed by the graphs of  $f$  and  $g$  is revolved about the horizontal line  $y = 2$ .
  - (c) Let  $h$  be the function given by  $h(x) = kx(1 - x)$  for  $0 \leq x \leq 1$ . For each  $k > 0$ , the region (not shown) enclosed by the graphs of  $h$  and  $g$  is the base of a solid with square cross sections perpendicular to the  $x$ -axis. There is a value of  $k$  for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of  $k$ .

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

## Question 7

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Under A Curve, Volume of Revolution – Disc Method, Volume of Revolution – Washer Method

Paper: Part A-Calc / Series: 2004-Form-B / Difficulty: Easy / Question Number: 1

1. Let  $R$  be the region enclosed by the graph of  $y = \sqrt{x - 1}$ , the vertical line  $x = 10$ , and the  $x$ -axis.
  - (a) Find the area of  $R$ .
  - (b) Find the volume of the solid generated when  $R$  is revolved about the horizontal line  $y = 3$ .
  - (c) Find the volume of the solid generated when  $R$  is revolved about the vertical line  $x = 10$ .

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)



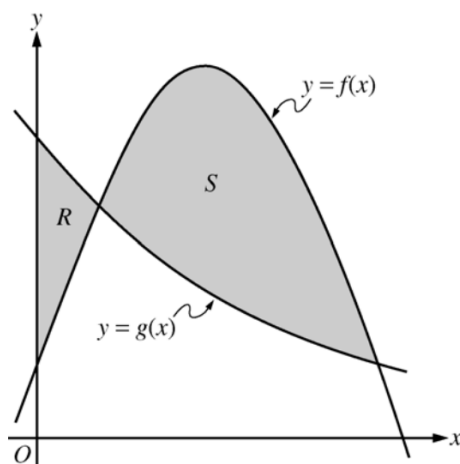
## Question 8

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method

Paper: Part A-Calc / Series: 2005 / Difficulty: Easy / Question Number: 1



- Let  $f$  and  $g$  be the functions given by  $f(x) = \frac{1}{4} + \sin(\pi x)$  and  $g(x) = 4^{-x}$ . Let  $R$  be the shaded region in the first quadrant enclosed by the  $y$ -axis and the graphs of  $f$  and  $g$ , and let  $S$  be the shaded region in the first quadrant enclosed by the graphs of  $f$  and  $g$ , as shown in the figure above.
  - Find the area of  $R$ .
  - Find the area of  $S$ .
  - Find the volume of the solid generated when  $S$  is revolved about the horizontal line  $y = -1$ .

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

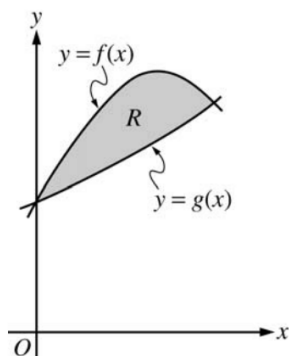
## Question 9

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Volume using Cross Sections, Integration - Area Between Curves, Volume of Revolution – Washer Method

Paper: Part A-Calc / Series: 2005-Form-B / Difficulty: Medium / Question Number: 1



1. Let  $f$  and  $g$  be the functions given by  $f(x) = 1 + \sin(2x)$  and  $g(x) = e^{x/2}$ . Let  $R$  be the shaded region in the first quadrant enclosed by the graphs of  $f$  and  $g$  as shown in the figure above.
  - (a) Find the area of  $R$ .
  - (b) Find the volume of the solid generated when  $R$  is revolved about the  $x$ -axis.
  - (c) The region  $R$  is the base of a solid. For this solid, the cross sections perpendicular to the  $x$ -axis are semicircles with diameters extending from  $y = f(x)$  to  $y = g(x)$ . Find the volume of this solid.

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

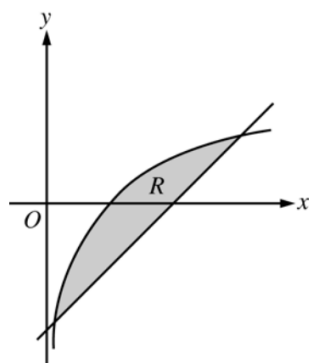
## Question 10

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method

Paper: Part A-Calc / Series: 2006 / Difficulty: Medium / Question Number: 1



1. Let  $R$  be the shaded region bounded by the graph of  $y = \ln x$  and the line  $y = x - 2$ , as shown above.
  - (a) Find the area of  $R$ .
  - (b) Find the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = -3$ .
  - (c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when  $R$  is rotated about the  $y$ -axis.

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

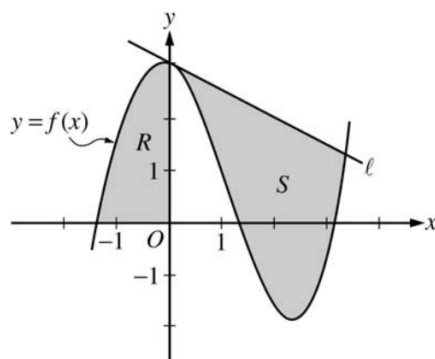
## Question 11

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Integration - Area Under A Curve, Integration - Area Between Curves, Tangents To Curves, Volume of Revolution – Washer Method

Paper: Part A-Calc / Series: 2006-Form-B / Difficulty: Easy / Question Number: 1



1. Let  $f$  be the function given by  $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$ . Let  $R$  be the shaded region in the second quadrant bounded by the graph of  $f$ , and let  $S$  be the shaded region bounded by the graph of  $f$  and line  $\ell$ , the line tangent to the graph of  $f$  at  $x = 0$ , as shown above.
  - (a) Find the area of  $R$ .
  - (b) Find the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = -2$ .
  - (c) Write, but do not evaluate, an integral expression that can be used to find the area of  $S$ .

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

## Question 12

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Under A Curve, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2007 / Difficulty: Somewhat Challenging / Question Number: 1

1. Let  $R$  be the region in the first and second quadrants bounded above by the graph of  $y = \frac{20}{1+x^2}$  and below by the horizontal line  $y = 2$ .
- (a) Find the area of  $R$ .
  - (b) Find the volume of the solid generated when  $R$  is rotated about the  $x$ -axis.
  - (c) The region  $R$  is the base of a solid. For this solid, the cross sections perpendicular to the  $x$ -axis are semicircles. Find the volume of this solid.

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

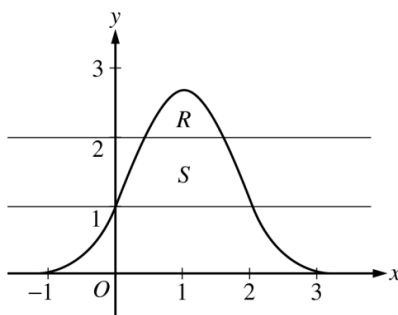
## Question 13

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method

Paper: Part A-Calc / Series: 2007-Form-B / Difficulty: Medium / Question Number: 1



1. Let  $R$  be the region bounded by the graph of  $y = e^{2x-x^2}$  and the horizontal line  $y = 2$ , and let  $S$  be the region bounded by the graph of  $y = e^{2x-x^2}$  and the horizontal lines  $y = 1$  and  $y = 2$ , as shown above.
  - (a) Find the area of  $R$ .
  - (b) Find the area of  $S$ .
  - (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = 1$ .

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

## Question 14

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2008-Form-B / Difficulty: Easy / Question Number: 1

1. Let  $R$  be the region in the first quadrant bounded by the graphs of  $y = \sqrt{x}$  and  $y = \frac{x}{3}$ .
- (a) Find the area of  $R$ .
  - (b) Find the volume of the solid generated when  $R$  is rotated about the vertical line  $x = -1$ .
  - (c) The region  $R$  is the base of a solid. For this solid, the cross sections perpendicular to the  $y$ -axis are squares. Find the volume of this solid.

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

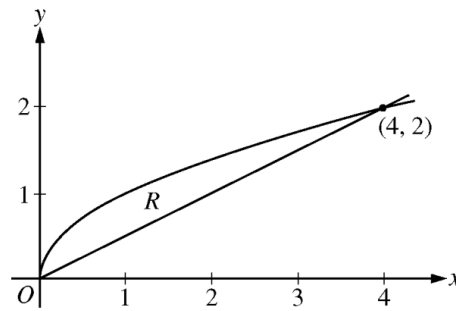
## Question 15

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume using Cross Sections, Volume of Revolution – Washer Method, Integration Technique - Harder Powers

Paper: Part B-Non-Calc / Series: 2009-Form-B / Difficulty: Easy / Question Number: 4



4. Let  $R$  be the region bounded by the graphs of  $y = \sqrt{x}$  and  $y = \frac{x}{2}$ , as shown in the figure above.
- Find the area of  $R$ .
  - The region  $R$  is the base of a solid. For this solid, the cross sections perpendicular to the  $x$ -axis are squares. Find the volume of this solid.
  - Write, but do not evaluate, an integral expression for the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = 2$ .

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)



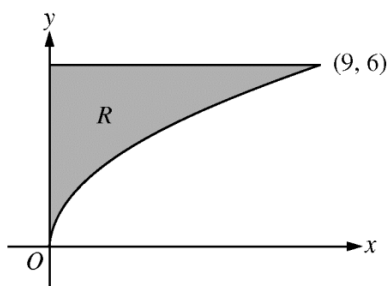
## Question 16

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part B-Non-Calc / Series: 2010 / Difficulty: Easy / Question Number: 4



4. Let  $R$  be the region in the first quadrant bounded by the graph of  $y = 2\sqrt{x}$ , the horizontal line  $y = 6$ , and the  $y$ -axis, as shown in the figure above.
- (a) Find the area of  $R$ .
  - (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = 7$ .
  - (c) Region  $R$  is the base of a solid. For each  $y$ , where  $0 \leq y \leq 6$ , the cross section of the solid taken perpendicular to the  $y$ -axis is a rectangle whose height is 3 times the length of its base in region  $R$ . Write, but do not evaluate, an integral expression that gives the volume of the solid.
- 

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

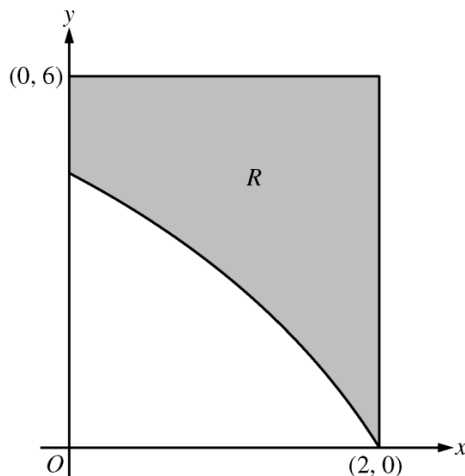
## Question 17

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2010-Form-B / Difficulty: Easy / Question Number: 1



1. In the figure above,  $R$  is the shaded region in the first quadrant bounded by the graph of  $y = 4\ln(3 - x)$ , the horizontal line  $y = 6$ , and the vertical line  $x = 2$ .
    - (a) Find the area of  $R$ .
    - (b) Find the volume of the solid generated when  $R$  is revolved about the horizontal line  $y = 8$ .
    - (c) The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a square. Find the volume of the solid.
- 

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

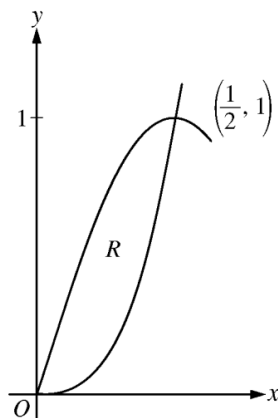
## Question 18

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration, Integration

Subtopics: Tangents To Curves, Integration - Area Between Curves, Integration Technique – Trigonometry, Volume of Revolution – Washer Method

Paper: Part B-Non-Calc / Series: 2011 / Difficulty: Easy / Question Number: 3



3. Let  $R$  be the region in the first quadrant enclosed by the graphs of  $f(x) = 8x^3$  and  $g(x) = \sin(\pi x)$ , as shown in the figure above.
- (a) Write an equation for the line tangent to the graph of  $f$  at  $x = \frac{1}{2}$ .
  - (b) Find the area of  $R$ .
  - (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = 1$ .
- 

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

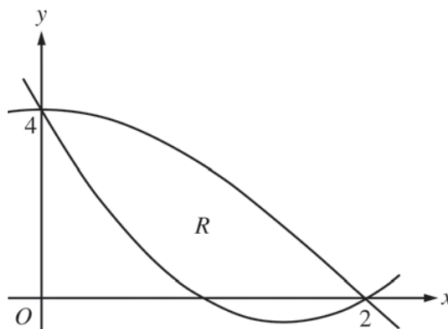
## Question 19

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections, Integration Technique – Trigonometry, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2013 / Difficulty: Medium / Question Number: 5



5. Let  $f(x) = 2x^2 - 6x + 4$  and  $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$ . Let  $R$  be the region bounded by the graphs of  $f$  and  $g$ , as shown in the figure above.

- Find the area of  $R$ .
- Write, but do not evaluate, an integral expression that gives the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = 4$ .
- The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

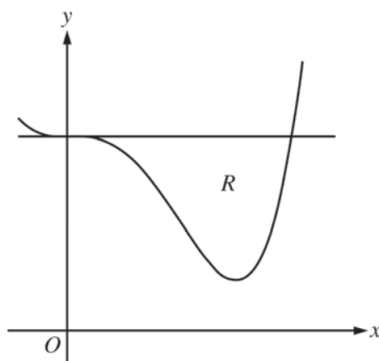
## Question 20

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Volume of Revolution – Washer Method, Volume using Cross Sections

Paper: Part A-Calc / Series: 2014 / Difficulty: Easy / Question Number: 2



2. Let  $R$  be the region enclosed by the graph of  $f(x) = x^4 - 2.3x^3 + 4$  and the horizontal line  $y = 4$ , as shown in the figure above.
- Find the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = -2$ .
  - Region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is an isosceles right triangle with a leg in  $R$ . Find the volume of the solid.
  - The vertical line  $x = k$  divides  $R$  into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value  $k$ .

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

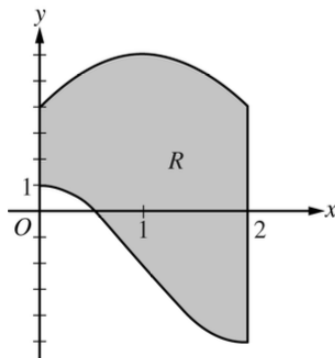
## Question 21

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume using Cross Sections, Volume of Revolution – Washer Method, Integration Technique – Trigonometry, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2019 / Difficulty: Medium / Question Number: 5



5. Let  $R$  be the region enclosed by the graphs of  $g(x) = -2 + 3 \cos\left(\frac{\pi}{2}x\right)$  and  $h(x) = 6 - 2(x - 1)^2$ , the  $y$ -axis, and the vertical line  $x = 2$ , as shown in the figure above.

(a) Find the area of  $R$ .

(b) Region  $R$  is the base of a solid. For the solid, at each  $x$  the cross section perpendicular to the  $x$ -axis has

area  $A(x) = \frac{1}{x + 3}$ . Find the volume of the solid.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = 6$ .

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)

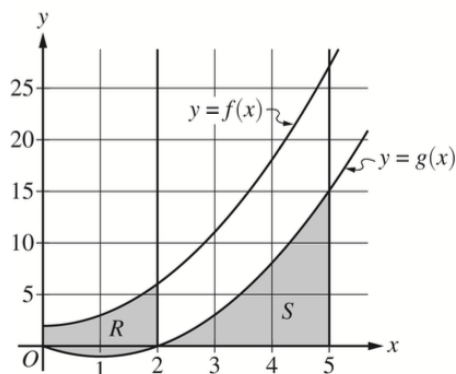
## Question 22

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Integration - Area Between Curves, Volume using Cross Sections, Volume of Revolution – Washer Method

Paper: Part B-Non-Calc / Series: 2024 / Difficulty: Medium / Question Number: 6



6. The functions  $f$  and  $g$  are defined by  $f(x) = x^2 + 2$  and  $g(x) = x^2 - 2x$ , as shown in the graph.
- (a) Let  $R$  be the region bounded by the graphs of  $f$  and  $g$ , from  $x = 0$  to  $x = 2$ , as shown in the graph. Write, but do not evaluate, an integral expression that gives the area of region  $R$ .
  - (b) Let  $S$  be the region bounded by the graph of  $g$  and the  $x$ -axis, from  $x = 2$  to  $x = 5$ , as shown in the graph. Region  $S$  is the base of a solid. For this solid, at each  $x$  the cross section perpendicular to the  $x$ -axis is a rectangle with height equal to half its base in region  $S$ . Find the volume of the solid. Show the work that leads to your answer.
  - (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region  $S$ , as described in part (b), is rotated about the horizontal line  $y = 20$ .

SCAN ME!



Mark Scheme

[View Online](#)

SCAN ME!



Written Mark Scheme

[View Online](#)